nical productivity of capital goods would withdraw current income until consumer goods were so scarce that an agio would be created. If we may interpret Boehm-Bawerk in this way, then his argument becomes quite similar to Fisher's.[10]

[10] See Arvidsson, "On the Reasons for a Rate of Interest," p. 27, for a still different interpretation of the independence of the third Reason, and for further discussion of the controversy.

IV

IRVING FISHER'S THEORY OF INTEREST

Despite the controversy between Boehm-Bawerk and Fisher, their theories of interest are constructed with similar building blocks, and Fisher dedicates his book, *The Theory of Interest,* to John Rae and Boehm-Bawerk, "Who laid the foundation on which I have endeavored to build." The two major elements in Boehm-Bawerk's theory, time preference and the productivity of capital, provide Fisher's first two "approximations": the third is risk.

Fisher presents his theory in three successive ways. First he gives it in words, next he presents it graphically, and finally he states it in systems of equations. I shall proceed directly to the graphic method, translating into words as I go, and following this by comments on the wider comprehensiveness which can be attained by use of equations instead of two-dimensional graphs.

GEOMETRIC PRESENTATION

As in general price theory, so also in the theory of interest it is useful to consider two aspects of the process by which equilibrium is approached. In the first place, individuals in competition take prices as given, and adjust quantities until their satisfactions are maximized. Secondly, the aggregate effect of these simultaneous individual acts may be described by an analysis of market equilibrium, where prices are not given, but adjust until total market supply and demand are equated. Fisher's analysis proceeds in a similar way, the goods in trade being present and future income, the price being the rate of interest. Figure I[1] illustrates both the adjustment of the individual, with price (interest rates) given, and the adjustment of the market, wherein the equilibrium rate of interest is determined.

In this figure, present income is represented on the x-axis, and "next

[1] Figures I through VIII will be found in a booklet inserted in the back of this book in order that they may be studied as the text is read without constant turning of pages.

year's income" is represented on the y-axis. The presumption is that the individuals X and Y start with claims to both present and future income in amounts indicated by the points O_x and O_y respectively. Thus X begins with 35 units of present income and a claim to 10 units next year, whereas Y starts with 58 units of current income and a claim to 20 next year.

Opportunity Curves

A brief illustration may help to clarify the economic meaning of both the "claims to income" just discussed and the "opportunities for investment" which will be examined in this section. If we own a mine that will soon be exhausted, we hold a claim to much present income and little future income. If we own a seedling forest, we hold a claim to no present income, but substantial future income. Likewise, if we hold a bond we hold a claim to future income.

There are various ways by which we may change the relation between our claims to present and to future income. (1) We may lend or borrow. Also we may buy or sell debt instruments, which is equivalent to lending or borrowing and will be so regarded throughout this analysis. (2) We may plant forests and manufacture capital goods, or we may cut the former and let the latter deteriorate. These activities whereby we increase or reduce the existing amounts of real earning assets may be regarded as investment and disinvestment. (3) Finally we may exchange ownership title to real earning assets, including share ownership in corporate enterprise. Fisher does not seem to be wholly clear about the treatment of this last type of transaction, and it will be best to make our first approach to his analysis by abstracting entirely from the exchange of claims to real assets. We then have two distinct types of transaction to examine. One is the act of borrowing and lending — or its equivalent, the sale and purchase of debt instruments — the geometric representation of which will be discussed in the following section. The other is the act of "real" investment or disinvestment, the presentation of which will be described here.

The investment opportunities, in this "real" sense, presented to X and Y are indicated by the "opportunity" curves passing through O_x and O_y respectively. By investing in capital goods, X could secure 18 additional units of income next year at a cost of only 7 units this year (moving from O_x to S_x); alternatively, he could sacrifice 15 units of current income and secure in exchange 28 additional units of income next year, assuming he places the 15 units in capital goods. Obviously X could invest the 15 units less wisely, securing in return some amount less than 28 units of next year's income; thus the OP curve is really an envelope indicating the most profitable of all possible investments. It surrounds an infinite number of less profitable investment curves, each of which repre-

sents a less attractive investment opportunity. The OP curve is concave to the origin, reflecting the assumption of diminishing returns to investment.[2] This assumption is illustrated in the example cited above by the fact that an investment of 7 will be rewarded with a return payment of 18, providing a gain of 11 against the investment of 7, or a "rate of return over cost" of 11/7, whereas an investment of 15 yields a rate of return over cost of "only" 13/15, i.e., (28 – 15)/15.

It may be noted that the average rate of return over cost for any investment is shown by deducting 1 from the slope of the line connecting the initial point (e.g., O_x) with the point where the investment stops (e.g., P_x). In this statement, and in all similar references to figure I throughout this book, we greatly simplify our expression by disregarding the negative sign of the slope of OP (and, correspondingly in other references, the slopes of AB and W curves). Illustrating our previous statement, then, the slope of a line connecting O_x with P_x is 28/15; the indicated rate of return for this investment has been shown to be (28 –15)/15 which equals (28/15) – 1. The fact that rates of return are indicated by slopes of the relevant lines *minus one* may be made intuitively plausible by noting that a slope of unity implies a gain of future income precisely equal to current sacrifice, and hence represents a zero rate of return over cost.

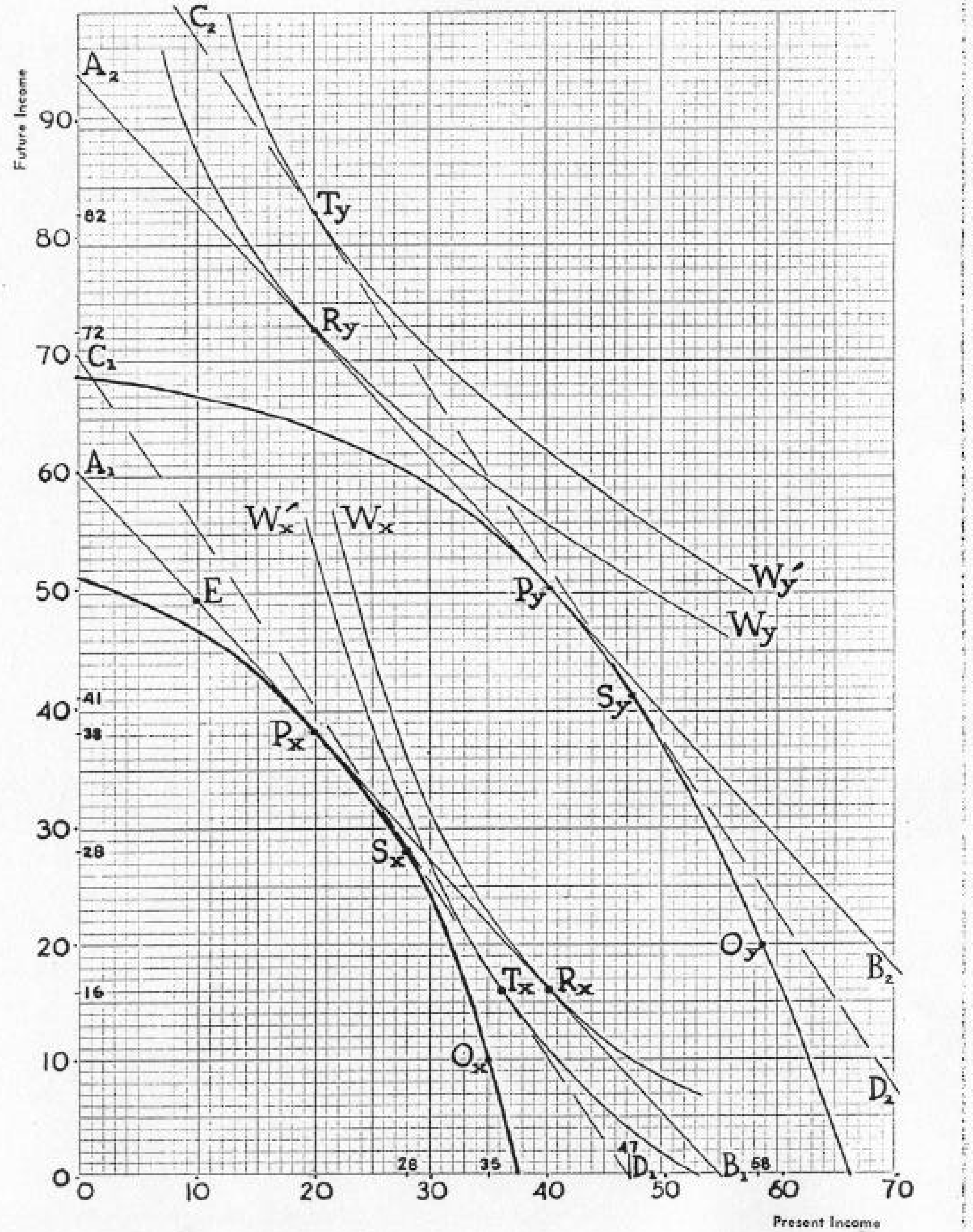
The figures just described are of course *average* rates of return. The *marginal* rate of return at any point is shown by the slope of the opportunity curve at that point, and is equal to that slope (again disregarding sign) minus one.

Interest Lines

A straight line may be drawn through any point on the graph to indicate any assumed rate of interest. Thus the AB lines[3] show that from any point on them a person can *lend* on the market 10 units of present income in return for a claim to 11 units next year (moving to the left and upward on the A_1B_1 line, as from E to A_1), or he can borrow by sacrificing 11 units of next year's income in return for 10 units of current income (moving down to the right on the A_1B_1 line, as from A_1 to E). The rate of interest in this case is 10 per cent (11 – 10)/10, which is read from the graph as the slope of the AB line minus one. Thus a 45 degree line represents zero rate of interest (repayment = original loan), and higher interest rates are shown by the increasing slope of the AB line above 45 degrees. A given interest rate may be represented by a family of an infinite number of AB lines of given slope.

[2] The legitimacy of this assumption is discussed on pp. 53–55.

[3] We shall hereafter refer to the lines whose "slope minus 1 = the interest rate" as the AB lines, without subscript. There are, of course, an infinite number of these for each rate of interest, all parallel to one another. A_1B_1 and A_2B_2 in figure I are two of


Future Income
Present Income
A₂
C₂
T_y
R_y
C₁
A₁
W_x'
W_x
E
P_y
W_y'
W_y
S_y
P_x
S_x
O_y
T_x
R_x
B₂
O_x
D₂
D₁
B₁
82
72
41
38
28
16
28
35
47
58
0
10
20
30
40
50
60
70
80
90

Willingness Curve

The third type of curve shown here is the "willingness" curve, the W curves shown convex to the origin being illustrative. These are essentially indifference curves. Every individual will have a whole family of such curves, each indicating combinations of present and future income that yield him equal satisfaction. I have drawn W_x and $W_{x'}$ to represent two of X's willingness curves, and I have similarly drawn W_y and $W_{y'}$ for Y.

Individual Equilibrium

As stated above, figure I may be used to illustrate either the equilibrium of the individual or that of the market. We shall begin with the former. Consider, for example, the adjustments made by citizen X, whose income claims at the start provide him with $35 this year and assurance of $10 next year. He can alter this income by either of two types of procedure: (1) he can borrow or lend at interest, or (2) he can engage in real investment or disinvestment. It is obvious that his optimum investment position will be shown by the point of tangency of the opportunity line with the interest line (A_1B_1). This follows because he can then attain any position on the tangent AB line by merely borrowing and lending; and any location he might attain on some other AB line (by stopping at a different point on the opportunity line) would be inferior to *some* position on the tangent AB line. Thus an AB line through S_x, parallel to A_1B_1 as I have drawn it, would lie closer to the origin throughout its length. From any point on the line through S_x an investor could, by moving to A_1B_1, either increase this year's income without loss of next years', or increase next year's income without loss of this year's, or increase each simultaneously. A simple way of stating this conclusion is to say that the market value of the individual's income stream will be maximized if he moves along the opportunity curve until he reaches the point of tangency with the interest line.[4]

Let us suppose that the interest rate is shown by the AB line. X will find it most profitable to invest in real capital goods until he reaches the position P_x, and Y will find it most profitable to invest along his opportunity line to the point P_y. But neither X nor Y is now satisfied with his distribution of income between present and future. X finds himself with a claim to only 20 units of income this year, when all his children are in college and he needs every cent he can muster; whereas he has a claim to 38 units "next year" when they will have graduated and his needs are small. He will therefore borrow until he reaches the highest possible indifference curve, which will, for familiar reasons, be the one tangent to the A_1B_1 line. He ends at R_x. As a result of investing and borrowing, X has increased his current income from 35 to 40, and he has simultaneously raised his future income from 10 to 16. No other plan for investment or for borrowing and lending could reach so high an indifference curve.

While the analytical problem must proceed by investing first, then borrowing and lending, there is obviously no implication regarding the chronology of the action taken. All that is suggested is that X borrows a total of 20 units (horizontal distance from P_x to R_x), invests 15 (horizontal distance from O_x to P_x), and dissaves 5 (spends 5 in excess of his current income; horizontal distance from O_x to R_x).

Similar interpretation shows that Y lends 20 (P_y to R_y), invests 18 (O_y to P_y), and saves 38 (O_y to R_y). Equilibrium is established for each individual, for he maximizes his satisfaction under the given conditions and tastes; it is established for the market because, at the given interest rates, lending by Y (20) precisely equals borrowing by X.

Market Equilibrium

We have now examined the adjustment of individuals in a market where an equilibrium rate of interest had already been established. We next observe the way in which the rate might adjust toward such an equilibrium if it were at first in disequilibrium. Suppose the situation is identical to that just described except that the initial rate is shown by the CD lines instead of by the AB lines. X will now wish to invest 7 (O_x to S_x), borrow 8 (S_x to T_x), and dissave 1 (O_x to T_x). Y, in the meantime, will want to invest 11 (O_y to S_y), lend 27 (S_y to T_y), and save 38 (O_y to T_y). The crucial point is that Y wishes to lend 27 and X wishes to borrow only 8. Assuming, in order to avoid problems of bilateral monopoly, that the market consists of many such individuals, it is clear that the rate of interest will be pushed down because of Y's desire to lend more than he can at going rates, and because X's desire to borrow far less than is available at going rates. The fall of the interest rate under these "forces" is recorded by the shift of the CD lines toward more nearly horizontal slope. When they reach the position of the AB lines, they move no further, for at this point the quantity which lenders wish to make available exactly equals the quantity that borrowers wish to secure.

[4] The fact that X would maximize the market value of his income by investing to the point P assuming the market rate of interest = 10% and OP is as drawn may be illustrated by comparing the value of this stream with that of alternative points on each side of P along OP. Using the familiar discount formula we may write:

$$V \text{ at P} = Y_0 + \text{Discounted Value of } Y_1 = Y_0 + \frac{Y_1}{1 + 10\%} = 20 + \frac{38}{1.1} = 54.54$$

$$V \text{ slightly left of P} = Y_0 + \frac{Y_1}{1.1} = 18 + \frac{40}{1.1} = 54.36$$

[illegible] Y_1 [illegible] 32 [illegible]

It will be noted that in Fisher's model as thus far presented (his first two approximations)[5] equilibrium requires that the marginal rate of return over cost equal the rate of interest, which in turn must also equal the marginal rate of time preference (the slope of the willingness curve minus one). The first of these conditions is parallel to the familiar proposition that in equilibrium investment will be carried to the point where the marginal efficiency of capital (Keynes' vocabulary) will be just equal to the rate of interest.[6] The second condition is essentially a definition of the psychology which lies behind the construction of a supply curve that shows saving as a function of the rate of interest.

By a slight transformation we can show that this equilibrium is also characterized by the equality of planned saving and investment, which Keynes regards as the "classical" theory of interest. This equivalence may be derived as follows. In Fisher's equilibrium, desired lending equals desired borrowing. But Y's desired lending equals his desired saving (horizontal O_y to $R_y = 38$) minus his desired investment (O_y to $P_y = 18$). X's desired borrowing, on the other hand, equals his desired investment (horizontal O_x to $P_x = 15$) plus his desired dissaving, which may be conveniently stated as "minus his desired (negative) saving" (horizontal O_x to $R_x = 5$). With these substitutions we may restate the condition of equilibrium as follows:

$$\text{Desired lending} \quad = \quad \text{Desired borrowing}$$
$$S_y\,(=38) - I_y\,(=18) = I_x\,(=15) - S_x\,(=-5)$$

By transposing we have:

$$S_y\,(=38) + S_x\,(=-5) = I_x\,(=15) + I_y\,(=18)$$

or

$$S\,(=33) = I\,(=33).$$

In short, Fisher's theory states that the rate of interest equates saving and investment, assuming these to be "desired" or "ex ante" magnitudes.

We may note also that this model takes income as given (presumably on the full-employment assumption), except to the extent that it is altered by the output flowing from real investment. The decision of persons to increase saving reduces interest rates, with the result that (1) the amount of desired saving is, typically, somewhat less than if rates had not fallen, and (2) the quantity invested increases. The same result can be shown by moving a saving curve to the right in a simplified loanable funds

[5] Discussion of the third approximation is presented on pp. 65–67.

[6] Keynes recognizes the identity explicitly, though it should be qualified by the important fact that Keynes' term is an "expected value." See J. M. Keynes, *The General Theory of Employment, Interest and Money* (New York: Harcourt, Brace, 1936), p. 140. For discussion of a context in which essential differences emerge between MEC and

model in which demand is represented by investment and supply by saving.

It may be noted in passing that this transformation from equality of lending and borrowing to equality of saving and investing slightly modifies Fisher's supply and demand curves by netting out dissaving and disinvestment. (Fisher's supply curve would include both saving and disinvestment, his demand curve would include both investment and dissaving.) The resulting interest rate is, of course, unchanged by this transformation.

NOTES ON ALGEBRAIC PRESENTATION

In most graphical presentations of economic problems, the confinement to two (or at most three) dimensions makes for difficulties in exposition which can be overcome by conversion to algebra. The behavioral assumptions and the market equalities that characterize the equilibrium described above provide the bases for a set of equations wherein Fisher derives a determinate equilibrium assuming n individuals instead of 2, and m time periods instead of only "this year" and "next year."[7] His resulting solutions provide him with not one but many interest rates, each being the rate that will rule during some future year. With the aid of Friedrich Lutz' analysis (see chapter xv below) one can then determine the rates on securities of different term, retaining Fisher's assumption of perfect foresight. This reduction of the system to algebra not only permits the solution of multidimensional problems, but it also removes the awkwardness which arises because the curves in the system will in fact shift as one moves toward equilibrium. Thus, for instance, changing interest rates will change a person's income if he borrows or lends extensively, and this will cause his indifference pattern to shift. Geometrically the problem is most awkward, algebraically it need not present any difficulty.

SOME PROBLEMS OF INTERPRETATION

Shape of OP Curve

A few problems which sometimes arise in the interpretation of this theory deserve comment. First, why is not the investment opportunity curve a straight line for the ordinary individual? Fisher explains the concavity toward the origin by the law of diminishing returns: "Every investment *in his farm* will have a variable decreasing return . . ."[8] But the question raised at once is, why not buy more land? In a perfectly competitive society such as that which underlies Fisher's model, will the *individual* face decreasing returns to scale as expansion takes place? The answer consistent with conventional theory would appear to be that managerial limi-

[7] The system of equations described here is presented in the appendix to this chapter.